CHAPTER SEVEN

VARIATION

Introduction:

This shows the relationship which exists between two variables or quantities.
- For example we may choose to consider the relationship between the number of gallons of petrol used by a travelling car and the distance travelled.
- This relationship or variation will be that, the greater the distance travelled, the greater will be the number of gallons of petrol used.

- Variation truly speaking is the same as proportion.

Types:

- Basically there are three types and these are: :

1. Direct variation.

2. Inverse variation.

3. Partial variation.

NB: Associated with each of these is what certain individuals refer to as joint variation

DIRECT VARIATION :

-This may also be referred to as direct proportion:

-Two variables or quantities such as x and y are said to vary directly, or are directly proportional if

1. both x and y increases at the same time e.g

X	2	3	4	5	6	7
Y	4	8	12	16	20	24

2. both values of x and y decrease at the same time. i.e X decrease as y decreases. e.g.

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X	100	80	60	40	20
Y	10	8	6	4	2

Now consider two variables x and y. If x varies directly as y, then we write
$x \propto y$, i.e x is directly proportional to y.

From $x \propto y$, in order to remove the proportional sign, we must introduce a constant,

i.e if $x \propto y$, $\Longrightarrow$ x = ky, where k = a constant, which may be referred to as the proportionality constant or the constant of proportionality.

Also if M varies directly as R, then $M \propto R \Longrightarrow M = K.R$, where K is a constant.

Q1. Two variables m and v are such that m varies directly as v. Given the constant as 10, calculate m when v = 20.

Solution

Since m varies directly as v

$\Longrightarrow m \propto v, \Longrightarrow m = kv$

Where k is a constant.

Since k = 10 $\Longrightarrow$ m = 10v.

When v = 20 $\Longrightarrow$ m = (10)(20) = 200.

Q2. The population, P of a nation is directly proportional to the birth rate, R. If the constant is 20, determine the population in million if R = 5.

Solution.

P varies directly as R

$\Longrightarrow p \propto R \Longrightarrow p = KR$,

Where k is a constant. Since k = 20 $\Longrightarrow$ p = 20 R.

When R = 5 $\Longrightarrow$ P = (20) (5) = 100

The population, P = 100 million.

Q3. The velocity v of a car is directly proportional to its mass m squared. Given the constant as 5 and the mass of the car as 60kg, calculate the velocity in m/s.

Solution.

V is directly proportional to m squared $\Rightarrow v \propto m^2$,

$\Rightarrow v = km^2$, where k is a constant.

Since k = 5 => $v = 5m.^2$

When m = 60 $\Rightarrow v = 5(60)^2$,

$\Rightarrow v = 5\ (3600) = 1800$,

=> V = 18000 m\s.

Q4. The circumference of a circle varies directly as it radius cubed. Determine the circumference in cm, if the radius is 3cm. Take the proportionality constant to be 4.

Solution

Let c = the circumference and r = the radius.

Since the circumference varies directly as the radius cubed

$\Rightarrow C \propto r^3 \Rightarrow c = kr^3$,

where k = the constant. Since r = 3cm and k = 4

$\Rightarrow c = (4)(3)^3 = (4)(27) = 108.$

The circumference = 108cm.

Q5. The density of a material is directly proportional to the square root of its mass. Given the variation constant as 5 and the mass as 25g, determine the density in g cm^{-3}.

Let d = density and m = mass.

Since the density is directly proportional to the square root of the mass

$\Rightarrow d \propto \sqrt{m}$,

$\Rightarrow d = k.\sqrt{m}$, where k = the constant. Since k = 5 and m =25 $\Rightarrow d = 5.\sqrt{25}$, $\Rightarrow d = (5)\ (5) = 25$,

$\Rightarrow d = 25\ cm^{-3}$

Q6. The speed of a vehicle varies directly as it mass. The speed also varies as its length. Find the speed in km/h, if the car is of mass 120kg and it is 50 m long. Take the variation constant to be 10.

Solution.

Let s = speed, m = mass and *l* = length.

1. The speed varies directly as the mass $\Longrightarrow s \propto m$…………..(1)

11. Also the speed varies directly as the length $\Longrightarrow s \propto l$………(2)

From 1 and 2 i.e $s \propto m$ and $s \propto l$

$\Longrightarrow s \propto m.l.$ Removing the proportion sign

$\Longrightarrow s = k.m.l.$

Bu since k = 10, m = 120 and l = 50

$\Longrightarrow s = (10)(120)(50) = 60{,}000$

$\Longrightarrow s = 60{,}000$ km/h.

Q7. The velocity of a car varies directly as its mass squared. It also varies directly as the length of the car. Given that the car has a length of 10m and a mass of 30kg, calculate its velocity in km/h. Take the proportionality constant as 200.

Solution

Let v = velocity, m = mass and l = length.

1. Since the velocity varies directly as the mass squared

$\Longrightarrow v \propto m^2$.

2. Also since the velocity varies directly as the length

$\Longrightarrow V \propto l.$

From 1. $V \propto m^2$ and from 2. $V \propto l$,=> $v \propto m^2 l \Rightarrow v = km^2 l.$

But m = 30, k = 200 and l = 10 .

From $v = km^2l \Longrightarrow v = (200)(30)^2(10)$,

$\Longrightarrow v = (200)(900)(10)$,

$\Longrightarrow v = 18\ 000\ 00$ km/h.

8. The distance d covered by a vehicle varies directly as its length, l cubed as well as the square root of its mass m. Given that when m = 9kg, l = 2m and d = 120km,

1. determine the value of the constant.

11. deduce an expression for d in terms of m, l and k, where k is the constant

111. determine the distance travelled by the car in kilometers, when it is 40m long and has a mass of 60kg.

Solution

1.Since d varies directly as l cubed $\Rightarrow$ d $\propto l^3$

11. Since d also varies directly as the square root of m

$\Rightarrow$ d $\propto \sqrt{m}$.

Now since d $\propto l^3$ and d $\propto \sqrt{m}$,

$\Rightarrow$ d $\propto l^3.\sqrt{m}$ => d = k.$l^3.\sqrt{m}$..

But d = 120 when l = 2 and m = 9, and since d = k $l^3\sqrt{m}$,

$\Rightarrow$120 = k(2)^3($\sqrt{9}$) = k(8)(3) = 24k,

$\Rightarrow$120 = 24k $\Rightarrow$ k = $^{120}/_{24}$ = 5.

$\therefore$ K = 5.

11. We have already noted that d = k$l^3\sqrt{m}$.

To deduce an expression for d in terms of k, l and , m, we only substitute the value of the constant (i.e. 5) into the just written expression.

From d = kl^3 $\sqrt{m}$ $\Rightarrow$d = 5$l^3\sqrt{m}$.

111. When l = 40m and m = 60kg,

$\Rightarrow$ d = 5 l^3 $\sqrt{m}$ $\Rightarrow$d = 5(40)^3 $\sqrt{60}$,

$\Rightarrow$ d = 5 (64000) (7.7),

$\Rightarrow$ d = 2,464 000.

9. The quantities d, w, l and T are such that d varies directly as w squared. Also d varies directly as l as well as the square root of T. Given that d = 32 when w = 2, l = 1 and T= 4,

a. deduce an expression for d in terms of k, w, l and T, where k is the constant.

b. calculate d when w = 1, l = 10 and T = 25.

c. deduce an expression for l in terms of the constant, T and w.

d. calculate l when d =20, T=25 and w =1

solution

i. Since d varies directly as w squared $\Rightarrow$ d $\propto w^2$.

ii. Since d varies directly as l $\Rightarrow$ d $\propto$ l.

iii. Since d also varies directly as the square root of T $\Rightarrow$ d $\propto \sqrt{T}$.

Now from d $\propto w^2$, d $\propto$ l and d $\propto \sqrt{T}$ $\Rightarrow$ d $\propto w^2.l.\sqrt{T}$,

$\Rightarrow$ d = k w^2 l $\sqrt{T}$. We then determine the value of the constant .

Now d = 32, when w = 2, l = 1 and T = 4.

Since d = kw^2 l $\sqrt{T}$

$\Rightarrow$32= $K(2)^2(1)(\sqrt{4})$,

$\Rightarrow$32 = k(4)(1)(2),

$\Rightarrow$32 = 8k $\Rightarrow$ k= 32/8, $\Rightarrow$ k = 4.

a. For an expression for d, d = $kw^2l\sqrt{T}$ $\Rightarrow$ d = $4w^2$ l $\sqrt{T}$.

b. To calculate d when w = 1, l = 10 and T = 25,

$\Rightarrow$ d = $4(1)^2(10)(\sqrt{25})$, $\Rightarrow$ d = 4(1)(10)(5) =200.

c. To deduce an expression for l, we must first consider the equation d = kw^2 l $\sqrt{T}$ and make l the subject.

From d = kw^2 l. $\sqrt{T}$, divide through using $kw^2\sqrt{T}$

$$=> \frac{d}{kw^2\sqrt{T}} = \frac{kw^2 l\sqrt{T}}{kw^2\sqrt{T}}$$

$$=> \frac{d}{kw^2\sqrt{T}} = I, \Rightarrow I = \frac{d}{kw^2\sqrt{T}}$$

Substituting k = 4

$\Rightarrow$ l = $\frac{d}{4w^2\sqrt{T}}$ which is an expression for l in terms of k, w and T.

d. When d =20, T = 25 and w = 1,

$\Rightarrow I = \dfrac{d}{4w^2\sqrt{T}}$ $\Rightarrow I = \dfrac{20}{4(1)^2(\sqrt{25})}$

$= \dfrac{20}{4(1)(5)} = \dfrac{20}{20} = 1,$

$\Rightarrow I = 1$..

Inverse variation:

This may also be referred to as inverse proportion. Two variables such as x and y exhibit or show this type of variation, when as one variable increases, the other decreases.

Examples.

(1)

X	100	90	80	70	60	50
Y	1	2	3	4	5	6

(2)

X	200	300	400	500	600
Y	20	19	18	17	16

If x varies inversely as y, we write $x \propto 1/y$.

In or In order to remove the proportional sign, we introduce a constant.

From i.e $x \propto 1/y \Rightarrow x = k.1/y$, =>

=> $x = k/y$, where k is a constant.

Also if M varies inversely as Q, then $M \propto \frac{1}{Q}, \Rightarrow M = K.\frac{1}{Q}$

$\Rightarrow M = K/Q$, where k is a constant.

Q1 The velocity V of a car varies inversely as its mass m. Calculate the velocity in m/s, given that the mass of the car is 60kg and the variation constant is 240.

Solution.

Since v varies inversely as m,

$\Rightarrow v \propto 1/m$, $\Rightarrow V = k.\ 1/m \Rightarrow V = k/m$.

If If k = 240 and m = 60

$\Longrightarrow v = \frac{240}{60} = 4 \Longrightarrow V = 4m/s$.